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NOMENCLATURE

a_0	speed of sound in undisturbed fluid
A, B	integration constants
d_{ij}	deformation rate tensor
h	parallel channel height
I_1, I_2	dissipation integrals (see eq. 22))
k	coefficient of heat conduction
K	vortex strength
L	length of vortex
M	Mach number
p	pressure
r	radial distance
r_0	viscous core radius
S	entropy
t	time
T	temperature
\vec{u}, u_i	velocity vectors
u_θ	tangential velocity component
u	streamwise velocity
U_0	peak jet velocity at the nozzle exit
W	parallel channel width
\vec{x}, x_i	position vector
x	streamwise direction
y	direction normal to parallel channel wall

δ_{ij}	Kronecker delta
λ	second coefficient of viscosity
η	$\partial u_\theta / \partial r$
ξ	$\partial u / \partial y$
ϕ	velocity potential
Φ	viscous dissipation function
Φ_R	radiative component of viscous dissipation function
Φ_I	incompressible approximation to dissipation function
ρ	density
ν	kinematic viscosity
μ	viscosity
$\vec{\omega}, \omega_{ij}$	vorticity
$\overline{(\quad)}$	time average of the quantity in parentheses
$(\quad)'$	time-dependent fluctuation of the quantity in parentheses

VORTICAL DISSIPATION IN TWO-DIMENSIONAL SHEAR FLOWS

W. Clifton Horne and Krishnamurty Karamcheti*

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SUMMARY

An exact expression is derived for the viscous dissipation function of a real homogeneous and isotropic fluid, which has terms associated with the square of vorticity, wave radiation, and dilatation. The implications of the principle of maximal dissipation rate, as expressed by Ziegler and others, are explored by means of this equation for a parallel channel flow and a cylindrical vortex flow. The consequences of a condition of maximum dissipation rate on the growth of disturbances in an unsteady, laminar shear layer are apparently consistent with predictions and observations of maximum growth rate of vortical disturbances. Finally, estimates of the magnitudes of several dissipative components of an unsteady vortex flow are obtained from measurements of a periodic wall jet.

INTRODUCTION

The exact conservation equations of mass, momentum, and energy, which govern the motion of a real fluid are complete, yet intractable for most flows of interest, particularly for cases of unsteady or turbulent flow. Attempts to obtain solutions for the time-averaged form of these equations involve additional difficulties in closure because of the average product terms. The nonlinear terms in these equations have required the imposition of simplifying assumptions and approximate models, depending upon the particular flow under investigation.

In addition to the conservation equations, the second law of thermodynamics, which requires a net increase in entropy for a real process, has been applied to the analysis of shock waves and other irreversible phenomena. The phenomenon of turbulence is apparently consistent with the second law, as many commonly observed flows exhibit states of increasing disorder in the streamwise direction. However, a wide variety of flows are characterized by the persistence of a global structure such as waves and vortex arrays in the absence of global boundary conditions. The organization of such flows, many examples of which are presented by Van Dyke (ref. 1), do not exhibit simple structural organization characterized by increasing disorder in the streamwise direction.

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Ziegler (refs. 2 and 3) and others (refs. 4-5) have proposed an extension of the second law, termed the principle of maximal rate of entropy production. This principle is presented alternatively as the principle of maximal dissipation rate, the principle of least dissipative stress, or a principle of least deformation rate. The principle states that for a small material "element" throughout which properties are uniform, the observed motion results in a maximum of the dissipation rate, i.e., the rate of entropy production. This is not to be confused with the maximum entropy method of information theory, in which the entropy is associated with the loss or lack of information and the maximization of this quantity leads to an optimal representation of the power spectrum of a finite sample set (ref. 6). Ziegler shows these principles to be the consequence of a thermodynamic orthogonality condition relating the dissipative stresses and strains. In a later paper, Ziegler (ref. 7) employs the maximum entropy principle to derive constitutive relations for heat conduction and deformation in linear elastic and thermoplastic solids, and in inviscid and viscous gases and liquids.¹

Zeigler derives the orthogonality principle from a tensor expansion of the dissipation function. The associated extremum principles are based on the conditions that, for a real substance, surfaces of constant dissipation rate in deformation rate space are strongly convex (i.e., a plane which is tangent to a dissipation surface intersects that surface at one and only one point), and that along any ray extending from the origin of deformation rate space, the dissipation rate increases in value faster than a linear function of the distance from the origin. Ziegler also distinguishes between three types of processes, which are described as elementary, compound, or complex, depending respectively on whether the process involves a single tensor, multiple tensors which are uncoupled and independent, or multiple tensors which are coupled and dependent. The particular case of a heat conducting, newtonian fluid is described as a compound process (ref. 2, p. 265), and the orthogonality condition is to be applied separately to the viscous and thermal dissipation terms, but not necessarily to their sum, which is the rate of entropy production.

If it can be determined that the maximum entropy principle applies on a global basis, this principle would have a significant role in the determination of overall structure in real flows. In this paper, we will derive an exact expression of the dissipation function of a homogeneous, isotropic, newtonian fluid which has terms associated with the square of vorticity, wave generation, and dilatation. The implications of the maximal-dissipation-rate principle are explored by means of this equation for a parallel channel flow and a cylindrical vortex flow. Areas in which there is consistency between the principle and the notions of the instability of laminar shear flows are discussed. Finally, estimates of the magnitudes of several dissipative components of an unsteady vortex flow are obtained from measurements of a periodic wall jet.

¹Related discussions are presented in references 8-12.

DISSIPATION FUNCTIONS FOR NEWTONIAN FLOW

The energy conservation equation may be expressed in terms of the fluid entropy as:

$$\rho T \frac{DS}{Dt} = \Phi + k \nabla^2 T \quad (1)$$

The dissipation function, for a newtonian fluid, Φ , is written by Ziegler (ref. 2) as:

$$\Phi = \lambda d_{ii} d_{jj} + 2\mu d_{ij} d_{ij} \quad (2)$$

where

$$d_{ij} = \frac{1}{2} \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right)$$

Jeffreys (ref. 13) rewrites the latter expression of equation (2) as:

$$\Phi = \lambda \frac{\partial u_i}{\partial x_i} \frac{\partial u_j}{\partial x_j} + \mu \left[\frac{1}{2} \omega_{ij} \omega_{ij} + 2 \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right] \quad (3)$$

where

$$\omega_{ij} = \left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j} \right)$$

Hinze (ref. 14) obtains a similar decomposition of the time-averaged dissipation function, which is valid only for homogeneous turbulence. The last term in (3), $(\partial u_i / \partial x_j)(\partial u_j / \partial x_i)$, appears in several aeroacoustic wave equations, such as that derived by Phillips (ref. 15). Another formulation is obtained by combining the terms of the equation:

$$\nabla \cdot \vec{F} - \frac{D\mathbf{M}}{Dt} + \mathbf{M} \nabla \cdot \vec{u} = 0 \quad (4)$$

where

$$\mathbf{M} = \frac{\partial \rho}{\partial t} + u_i \frac{\partial \rho}{\partial x_i} + \rho \frac{\partial u_i}{\partial x_i} \quad (5)$$

$$\vec{F} = \rho \frac{\partial u_j}{\partial t} + \rho u_i \frac{\partial u_j}{\partial x_i} + \frac{\partial p}{\partial x_j} - \frac{\partial \tau'_{ij}}{\partial x_i} \quad (6)$$

$$\tau'_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \delta_{ij} \lambda \frac{\partial u_i}{\partial x_i}$$

$\mathbf{M} = \mathbf{\dot{F}} = 0$ for a fluid free of sources of mass or momentum. By expanding the terms of equation (4), we obtain:

$$\frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} = -\frac{1}{\rho} \left(\frac{\partial^2 p}{\partial x_j \partial x_j} - u_i u_j \frac{\partial^2 \rho}{\partial x_i \partial x_j} - 2u_j \frac{\partial^2 \rho}{\partial x_j \partial t} - \frac{\partial^2 \rho}{\partial t^2} \right) + \frac{1}{\rho} \frac{\partial^2 \tau'_{ij}}{\partial x_i \partial x_j} + \frac{\partial u_i}{\partial x_i} \frac{\partial u_j}{\partial x_j} \quad (7)$$

By substituting equation (7) into equation (3), the desired form is obtained:

$$\begin{aligned} \Phi = & \frac{2\mu}{\rho} (2\mu + \lambda) \frac{\partial^2}{\partial x_j \partial x_j} \left(\frac{\partial u_i}{\partial x_i} \right) + (\lambda - 2\mu) \frac{\partial u_i}{\partial x_i} \frac{\partial u_j}{\partial x_j} \\ & - \frac{2\mu}{\rho} \left(\frac{\partial^2 p}{\partial x_j \partial x_j} - u_i u_j \frac{\partial^2 \rho}{\partial x_i \partial x_j} - 2u_j \frac{\partial^2 \rho}{\partial x_j \partial t} - \frac{\partial^2 \rho}{\partial t^2} \right) + \frac{\mu}{2} \omega_{ij} \omega_{ij} \end{aligned} \quad (8)$$

This exact expression accounts for three important mechanisms of energy dissipation in real flows: (1) irreversible expansion or compression, (2) generation and radiation of sound or shock waves, and (3) generation of vorticity. According to the second law of thermodynamics, the overall entropy should increase for a real process, which implies a positive definite value for Φ . An extremum condition, as proposed by Ziegler on the rate of entropy production is a stronger constraint which may guide the determination of the distribution of energy dissipated by the various mechanisms.

The radiative dissipation term:

$$\Phi_R = -\frac{2\mu}{\rho} \left(\frac{\partial^2 p}{\partial x_j \partial x_j} - u_i u_j \frac{\partial^2 \rho}{\partial x_i \partial x_j} - 2u_j \frac{\partial^2 \rho}{\partial x_j \partial t} - \frac{\partial^2 \rho}{\partial t^2} \right) \quad (9)$$

may be simplified for comparison with other wave equations by first assuming an isentropic relationship between the pressure and density, and then examining the wave generation in flows with negligible velocity in all but one direction:

$$\rho' = \frac{p'}{a_0^2}, \quad u_1 = U \gg u_2, u_3; \quad M = \frac{U}{a_0}$$

For these conditions:

$$\Phi_R \approx -\frac{2\mu}{\rho} \left[(1 - M^2) \nabla^2 p' - 2 \frac{M}{a_0} \frac{\partial^2 p'}{\partial x_1 \partial t} - \frac{1}{a_0^2} \frac{\partial^2 p'}{\partial t^2} \right] \quad (10)$$

This expression may be compared with the compressible, unsteady, potential flow equation:

$$(1 - M^2) \nabla^2 \phi - \frac{2M}{a_0} \frac{\partial^2 \phi}{\partial x \partial t} - \frac{1}{a_0^2} \frac{\partial^2 \phi}{\partial t^2} = 0 \quad (11)$$

For waves of acoustic intensity, ϕ should be negligible except in the sound-producing region. For shock waves, the isentropic pressure-density relationship is invalid.

The principle of maximum dissipation rate is established for elemental volumes that contain a substance of uniform material properties and velocity. We now seek to determine if the principle holds on a global basis for selected flows. As with other physical principles or laws, the principle is validated by observations of consistency with real flows.

ENTROPY CONSIDERATIONS IN LOW-SPEED PARALLEL AND CYLINDRICAL SHEAR FLOWS

For incompressible, uniform-density flows, the expression (eq. (8)) for the dissipation function reduces to:

$$\Phi_I = - \frac{2\mu}{\rho_0} \nabla^2 p + \mu \omega^2 \quad (12)$$

According to the principle of maximum dissipation rate, we seek to find the extrema of the volume integral of equation (12) for selected familiar flows. For parallel channel flow, the mass and momentum equations reduce to:

$$\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} = - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\mu}{\rho_0} \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial y} = 0$$

For this case, the term in the dissipation function (eq. (12)) associated with the Laplacian of the pressure is zero, as is seen by taking the x -derivative of the momentum equation and by using the continuity equation:

$$\frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} \right)^0 = - \frac{1}{\rho_0} \frac{\partial^2 p}{\partial x^2} + \frac{\mu}{\rho_0} \frac{\partial^2}{\partial y^2} \left(\frac{\partial u}{\partial x} \right)^0 \quad (13)$$

Therefore,

$$\frac{\partial^2 p}{\partial x^2} = 0 \quad (14)$$

The volume integral of the dissipation function is now given by:

$$\int_V \Phi_I dv = \mu \int_V \omega^2 dv = \mu x W \int_{y_1}^{y_2} \left(\frac{\partial u}{\partial y} \right)^2 dy \quad (15)$$

where x and W are the length and width, respectively, of the channel.

To find the extrema of the preceding integral, we seek solutions of the Euler-Lagrange differential equation (see ref. 16)

$$\frac{\partial \Phi}{\partial u} - \frac{d}{dy} \left(\frac{\partial \Phi}{\partial \xi} \right) = 0 ; \quad \xi = \frac{\partial u}{\partial y} \quad (16)$$

For this example:

$$0 - \frac{d}{dy} \left(\frac{\partial \Phi}{\partial \xi} \right) = -2 \frac{d^2 u}{dy^2} = 0 \quad (17)$$

The dissipation extrema are found for the linear profile:

$$u(y) = Ay + B$$

Thus the linear velocity profile of a parallel channel flow corresponds to a dissipation extremum. This profile is observed when the channel walls are in relative motion and there is no pressure gradient in the channel.

Another simple example is that of a cylindrical rotational flow, in which the tangential velocity and pressure are functions only of the radius from the origin. Hence:

$$\vec{u} = \{u_r, u_\theta, u_z\} ; \quad u_r = u_z = 0$$

The radial pressure gradient is given by the radial momentum equation:

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{u_\theta^2}{r}$$

The Laplacian of the pressure is written as:

$$\nabla^2 p = \frac{1}{r} \frac{d}{dr} \left(r \frac{dp}{dr} \right) = \frac{1}{r} \frac{d}{dr} (\rho u_\theta^2) \quad (18)$$

The vorticity is given by:

$$\vec{\omega} = \vec{\nabla} \times \vec{u} = \frac{1}{r} \frac{d}{dr} (ru_\theta) = \frac{du_\theta}{dr} + \frac{u_\theta}{r} \quad (19)$$

$$\omega^2 = \left(\frac{du_\theta}{dr} \right)^2 + \frac{2}{r} u_\theta \frac{du_\theta}{dr} + \frac{u_\theta^2}{r^2} \quad (20)$$

We seek the extrema of the integral:

$$\int_V \Phi \, dv = - \frac{2\mu}{\rho} 2\pi L \int_0^\infty v^2 p r \, dr + \mu 2\pi L \int_0^\infty \omega^2 r \, dr \quad (21)$$

where L is the length of the vortex. The first integral on the right side vanishes, since from equation (18):

$$\begin{aligned} \int_0^\infty v^2 p r \, dr &= \int_0^\infty \frac{1}{r} \frac{d}{dr} (u_\theta)^2 r \, dr \\ &= \int_0^\infty \frac{d}{dr} (u_\theta^2) \, dr = u_\theta^2 \Big|_0^\infty \end{aligned} \quad (22)$$

and

$$u_\theta(0) = u_\theta(\infty) = 0$$

for any realizable flow. Then:

$$\int_V \Phi \, d\tau = 2\pi L \int_0^\infty \left[\left(\frac{du_\theta}{dr} \right)^2 + \frac{2}{r} u_\theta \frac{du_\theta}{dr} + \frac{u_\theta^2}{r^2} \right] r \, dr$$

The extrema are obtained as solutions of the Euler-Lagrange equation:

$$\frac{\partial \Phi}{\partial u_\theta} + \frac{d}{dr} \left(\frac{\partial \Phi}{\partial \eta} \right) = 0 ; \quad \eta = \frac{du_\theta}{dr} \quad (23)$$

$$\Phi = r \left(\frac{du_\theta}{dr} \right)^2 + 2u_\theta \frac{du_\theta}{dr} + \frac{u_\theta^2}{r}$$

$$\frac{\partial \Phi}{\partial u_\theta} = 2 \frac{du_\theta}{dr} + 2 \frac{u_\theta}{r} \quad (24)$$

Then

$$\begin{aligned}\frac{\partial \Phi}{\partial \eta} &= 2r \frac{du_{\theta}}{dr} + 2u_{\theta} \\ \frac{d}{dr} \left(\frac{\partial \Phi}{\partial \eta} \right) &= 2 \frac{du_{\theta}}{dr} + 2r \frac{d^2 u_{\theta}}{dr^2} + 2 \frac{du_{\theta}}{dr}\end{aligned}\quad (25)$$

By substituting equations (24) and (25) into (23), we obtain:

$$r^2 \frac{d^2 u_{\theta}}{dr^2} + r \frac{du_{\theta}}{dr} - u_{\theta} = 0$$

This is recognized as Euler's equation, with solutions:

$$u_{\theta} \propto r, r^{-1}$$

These two solutions represent respectively the core and the outer-potential region of a Lamb vortex:

$$\begin{aligned}u_{\theta} &= Kr \quad \text{for } 0 < r < r_0 \quad \{\text{viscous core}\} \\ u_{\theta} &= \frac{Kr_0^2}{r} \quad \text{for } r_0 < r < \infty \quad \{\text{inviscid outer flow}\}\end{aligned}$$

The two solutions represent minimum and maximum dissipation conditions in the same flow field. Unlike the previous example for the parallel channel flow, there are many solutions for the cylindrical vortex, including the Taylor vortex:

$$u_{\theta} = \frac{Kr}{t^2} \exp\{-r^2/4vt\}$$

The flow configuration obtained by maximum-dissipation considerations, with a viscous inner-flow region surrounded by an inviscid outer flow, is consistent with the basic notions of boundary layer theory. Further study is needed to determine the global validity of the dissipation principle for general laminar flows and for steady, high-speed flow. Another aspect of the dissipation principle (to be considered later) is the applicability of the principle, on a time-averaged basis, in the determination of the overall structure of unsteady and turbulent flow.

DISSIPATION CONSIDERATIONS IN UNSTEADY, TWO-DIMENSIONAL FLOWS

The maximum entropy principle is of considerable utility if it can be applied as a global constraint for complex flows, and further, if the principle can be applied in a time-averaged formulation. Neither of these approaches has been validated, and therefore both warrant further investigation.

The application of extremum conditions of entropy or dissipation rate to unsteady flows may be facilitated by considering the mean and fluctuating components of the dissipation function:

$$p = \bar{p} + p' ; \quad \omega = \bar{\omega} + \omega'$$

$$\Phi_I = - \frac{2\mu}{\rho} \nabla^2 (\bar{p} + p') + \mu (\bar{\omega} + \omega')^2 \quad (26)$$

$$\bar{\Phi}_I = - \frac{2\mu}{\rho} \nabla^2 \bar{p} + \mu [(\bar{\omega})^2 + \overline{\omega'^2}]$$

We may use this expression to interpret the small and large disturbance motions in two-dimensional flow. A large number of parallel shear flows such as the boundary-layer, the free-shear-layer, and the wall-jet flow have been successfully analyzed for stability to small disturbances via the Orr-Sommerfeld equation. The stability characteristics for these flows are found to be strongly associated with the mean velocity profile of the flow. For velocities which exceed a critical Reynolds number, small disturbances of the frequency corresponding to the maximum amplification rate are predicted to grow exponentially with downstream distance until the magnitude of the disturbances exceeds the level permissible in a linearized analysis. Within the small-disturbance region, the mean velocity profile remains unchanged, and the small disturbances take the form of convecting vortical motions. Experimental studies of various flows confirm that the frequencies predicted for maximum-vortical-disturbance growth rate correspond to the observed frequencies of unforced fluctuations. From equation (26), it appears that the maximum-vortical-disturbance growth rate corresponds to a maximal dissipation condition, since the contribution from the pressure terms is negligible and the contribution from the mean vorticity terms is independent of disturbance frequency.

Downstream of the small-disturbance, exponential-growth region, many parallel shear flows exhibit a comparably sized region of periodic large-vortical disturbances. In this region, the mean velocity profile is significantly altered with respect to the profile in the small-disturbance region. Eventually, the distinct vortex structures merge or break up into random, three-dimensional motions.

Figure 1 illustrates the periodic vortex motion of a two-dimensional wall jet flow. Figure 1(a) shows a phase-averaged schlieren visualization obtained by lightly heating the nozzle flow. Figure 1(b) shows the phase-averaged velocity field, referenced to the convecting vortices, and Figure 1(c) depicts the corresponding vorticity field obtained with a central-differenced curl of the velocity field. The vorticity is normalized with respect to the maximum exit velocity and the nozzle width. The measurements were obtained from a single x-wire velocity probe by sampling the probe output at regular phase intervals as determined by a fixed pressure-transducer in the wall (ref. 17). These measurements were further processed to obtain estimates of the mean and fluctuating components of the dissipation field.

Figures 2, 3, and 4 show contour plots of the respective distributions of square mean velocity, \bar{w}^2 , mean square of fluctuating vorticity, \bar{w}'^2 , and total mean square vorticity, $\bar{w}^2 + \bar{w}'^2$. Note that only the periodic component of the fluctuating vorticity is resolved with the measurement scheme. The schlieren photograph of the flowfield in figure 1 reveal the existence of irregular, convecting vortex structures beyond the region indicated by the measurements.

An overall view of the dissipative structure of the wall jet is gained by examining the variation of the dissipation integral with downstream distance. Here:

$$\int_V \Phi \, d\tau = \mu W \int_0^{x/h} \int_0^\infty \omega^2 \, dy \, dx' = \mu W x I_2\left(\frac{x}{h}\right) \quad (27)$$

where

$$I_2\left(\frac{x}{h}\right) = \int_0^{x/h} I_1\left(\frac{x'}{h}\right) d\left(\frac{x'}{h}\right)$$

and

$$I_1\left(\frac{x}{h}\right) = \int_{y_1}^{y_2} \omega^2 \left(\frac{h}{U_0}\right)^2 d\left(\frac{y}{h}\right)$$

Distributions of I_1 and I_2 are shown in figures 5 and 6, respectively. The value for I_1 is nearly constant throughout the wall jet until the end of the small-disturbance region, where it decreases by about 25%. The value of I_1 returns to about its initial level within the large-disturbance region. The overall vortical dissipation within the measuring region is approximately 1.1% of the kinetic energy supply from the nozzle. The total-acoustical-power output of the flow was estimated from reference 18 to be 0.02% of the kinetic energy rate.

These measurements demonstrate a potential method of directly measuring dissipative terms in an unsteady flow. A direct extension of this method may be applied to experimentally determine a relationship between overall dissipation and variable parameters, such as forcing frequency, and to experimentally search for dissipation extrema.

CONCLUSIONS

An exact equation has been derived for the dissipation function of a homogeneous, isotropic, newtonian fluid, with terms associated with fluid dilatation, wave radiation, and square vorticity. By using entropy extremum principles as given by Ziegler, simple flows such as the incompressible channel flow and the cylindrical Lamb vortex are identified as extremal configurations. The implications of the maximum dissipation conditions are found to be consistent with the principal notions of stability of parallel shear flows. The measurements of a typical periodic shear flow, the rectangular wall jet, show that direct measurement of the dissipation

terms is possible and that for this particular flow, the dissipation rate is nearly constant along the length of the jet.

Further experimental and theoretical analyses are required to determine the global validity of entropy extremum principles with regard to complex flows and to determine the applicability of time-averaged analyses. The limited observations obtained thus far suggest further applications to the analysis of turbulent, sound-generating, and shock-producing flows.

REFERENCES

1. Van Dyke, M. D.: An Album of Fluid Motion. Parabolic Press, 1983.
2. Ziegler, H.: An Introduction to Thermomechanics. North-Holland Publ. Co., 1977, pp. 253-269.
3. Ziegler, H.: A Possible Generalization of Onsager's Theory. H. Parkus and L. I. Sedov, eds., IUTAM Symp. on Irreversible Aspects of Continuum Mechanics, Julius Springer (Berlin), 1965.
4. Onsager, L.: Reciprocal Relations in Irreversible Processes. Phys. Rev., vol. 37, no. 2, 1931, p. 405, and vol. 38, no. II, 1931, p. 76.
5. Biot, M. A.: Variational Principles in Irreversible Thermodynamics with Application to Viscoelasticity. Phys. Rev., vol. 97, 1955, p. 1463.
6. Lacoss, R. T.: Data Adaptive Spectral Analysis Methods. Geophysics, vol. 36, no. 4, Aug. 1971, pp. 661-675.
7. Ziegler, H.: Constitutive Equations Derived from the Free Energy and the Dissipation Functions. Developments in Mechanics, vol. 13, Proceedings of the 19th Midwestern Mechanics Conference, 1985.
8. Ziegler, H.: Discussion of Some Objections to Thermodynamic Orthogonality. Ingenieur-Archiv, vol. 50, 1981, pp. 149-164.
9. Valanis, K. C.; and Komkov, V.: Irreversible Thermodynamics from the Point of View of Internal Variable Theory/A Lagrangian Formulation. Archinum Mechaniki Stosowanej, vol. 32, no. 1, 1980, pp. 33-58.
10. Paltridge, G. W.: Thermodynamic Dissipation and the Global Climate System. Royal Meteorological Society Quarterly Journal, vol. 107, July 1981, pp. 531-547.
11. Biot, M. A.: New Variational-Lagrangian Irreversible Thermodynamics with Application to Viscous Flow, Reaction-Diffusion, and Solid Mechanics. Advances in Applied Mechanics, vol. 24, 1984, pp. 1-91.
12. Liv, I-Shih: Method of Lagrange Multipliers for Exploitation of the Entropy Principle. Archive for Rational Mechanics and Analysis, vol. 46, no. 2, 1972, pp. 131-148.
13. Jeffreys, H.: Cartesian Tensors. Cambridge Univ. Press, 1974, pp. 84-89.
14. Hinze, J. O.: Turbulence. 2nd ed., McGraw-Hill, 1975.

15. Phillips, O. M.: On the Generation of Sound by Supersonic Turbulent Shear Layers. J. Fluid Mech., vol. 9, no. 1, 1960, p. 1.
16. Weinstock, R.: Calculus of Variations. Dover Publications, Inc., 1974, New York.
17. Horne, W. C.: Measurements of Large Scale Disturbances in Rectangular Wall Jets. AIAA Paper 84-2314, AIAA/NASA 9th Aeroacoustics Conf., Oct. 15-17, 1984, Williamsburg, Va., 1984.
18. Horne, W. C.; Joncouski, G.; and Karamcheti, K.: A Survey of the Acoustic Field of a Rectangular Wall Jet. AIAA Paper 81-2043, AIAA 7th Aeroacoustics Conf., Oct. 5-7, 1981, Palo Alto, Calif., 1981.

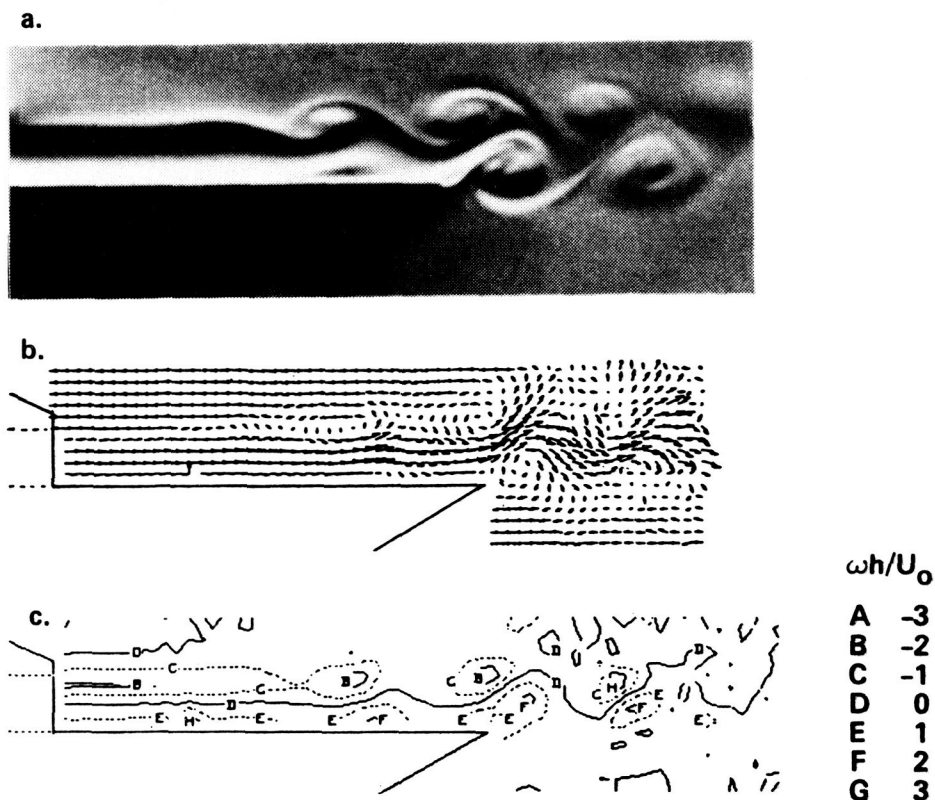
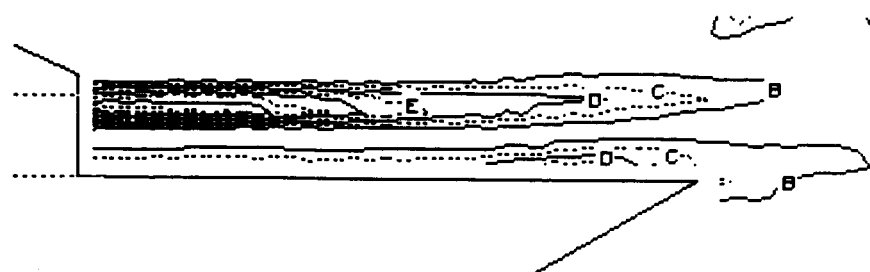
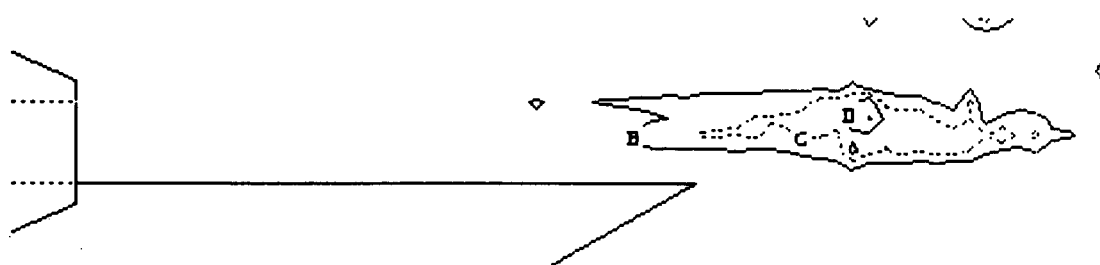


Figure 1.- Phase averaged velocity field and flow visualization. (a) Phase-averaged flow visualization; (b) velocity vectors (relative to convecting vortices; and (c) vorticity contours. Conditions: nozzle width, $h = 0.508$ cm; nozzle aspect ratio = 20; wall length, $L = 3.81$ cm; parabolic velocity profile at nozzle exit with maximum exit velocity, $U_0 = 13.85$ m/sec; tone frequency, $f = 600$ Hz.



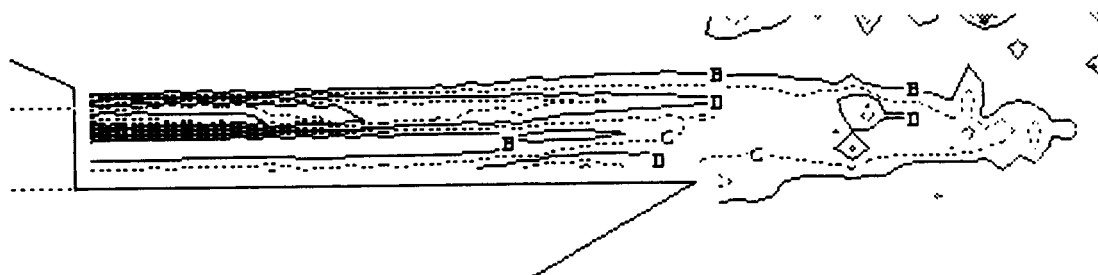
LABEL	CONTOUR VALUE OF
	$\bar{\omega}^2 \frac{h^2}{U_M^2}$
A	0
B	0.5
C	1.0
D	1.5
E	2.0
F	2.5

Figure 2.- Contour plot of the square of the mean vorticity distribution, $\bar{\omega}^2 h^2 / U_o^2$.



LABEL	CONTOUR VALUE OF
	$\overline{\omega'^2} \frac{h^2}{U_M^2}$
A	0
B	0.5
C	1.0
D	1.5
E	2.0

Figure 3.- Contour plot of the mean square of the fluctuating vorticity distribution, $\overline{(\omega')^2} h^2 / U_o^2$.



LABEL	CONTOUR VALUE OF
	$\overline{\omega^2} \frac{h^2}{U_M^2}$
A	0
B	0.5
C	1.0
D	1.5
E	2.0
F	2.5

Figure 4.- Contour plot of the mean-total-square vorticity distribution,

$$\overline{(\omega)^2} h^2 / U_o^2.$$

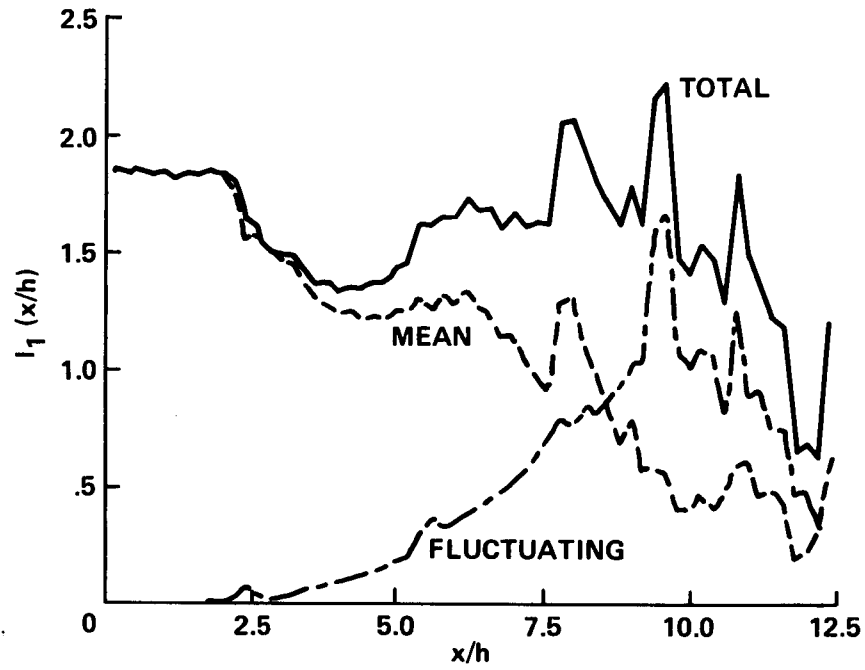


Figure 5.- Distribution of the first dissipation integral: $I_1(x/h)$ vs. x/h , where

$$I_1\left(\frac{x}{h}\right) = \int_{y_1}^{y_2} \omega^2 \left(\frac{h}{U_0}\right)^2 d\left(\frac{y}{h}\right)$$

The nonzero vorticity region is enclosed by $y_1 < y < y_2$.

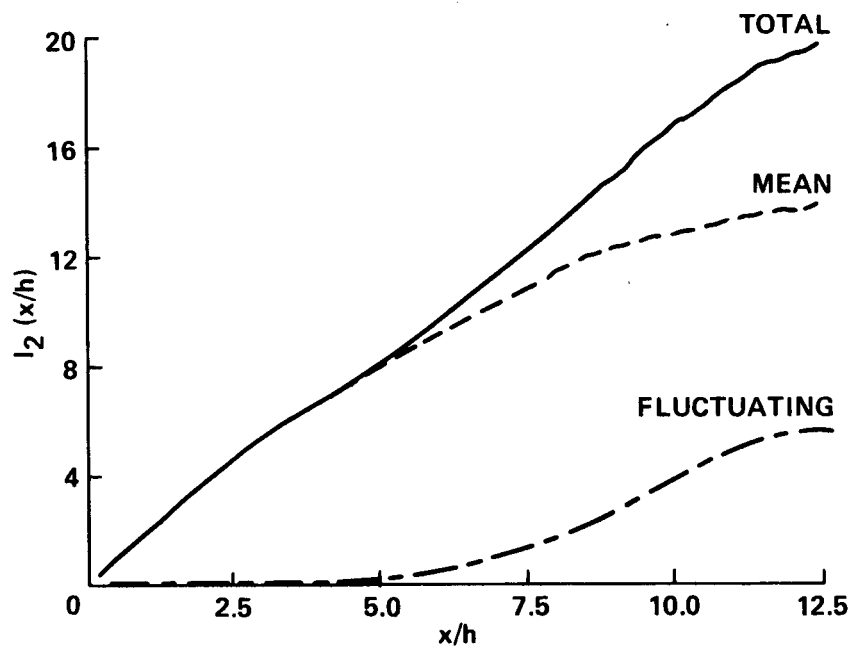


Figure 6.- Distribution of the second dissipation integral: $I_2(x/h)$ vs. x/h , where

$$I_2\left(\frac{x}{h}\right) = \int_0^x \int_{y_1}^{y_2} \omega^2 \left(\frac{h^2}{U_0^2}\right) d\left(\frac{y}{h}\right) d\left(\frac{x'}{h}\right)$$

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16. Abstract An exact expression is derived for the viscous dissipation function of a real homogeneous and isotropic fluid, which has terms associated with the square of vorticity, wave radiation, and dilatation. The implications of the principle of maximal dissipation rate, as expressed by Ziegler and others, are explored by means of this equation for a parallel channel flow and a cylindrical vortex flow. The consequences of a condition of maximum dissipation rate on the growth of disturbances in an unsteady, laminar shear layer are apparently consistent with predictions and observations of maximum growth rate of vortical disturbances. Finally, estimates of the magnitudes of several dissipative components of an unsteady vortex flow are obtained from measurements of a periodic wall jet.					
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